The Matching Problem in General Graphs is in QUASI-NC

Jakub Tarnawski

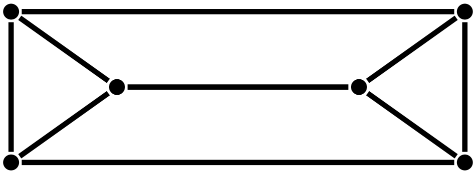
joint work with Ola Svensson

ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

October 16, 2017

Perfect matching problem

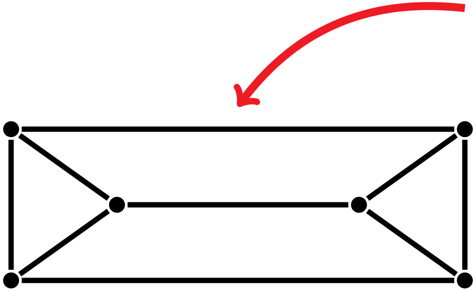
Given a graph, can we pair up all vertices using edges?



Perfect matching problem

Given a graph, can we pair up all vertices using edges?

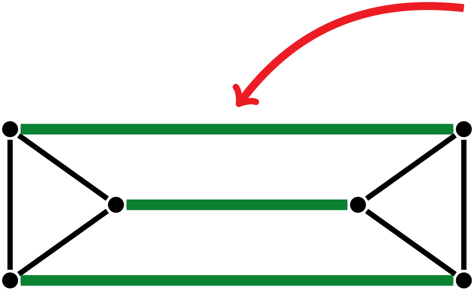
very tough instance:
graph is non-bipartite!



Perfect matching problem

Given a graph, can we pair up all vertices using edges?

very tough instance:
graph is non-bipartite!

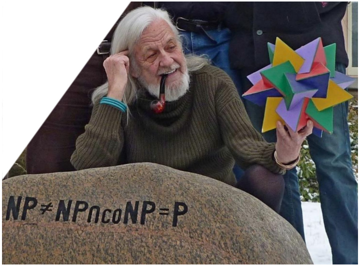


Perfect matching problem

Benchmark problem in computer science

Algorithms:

- bipartite: Jacobi [XIX century, weighted!]
- general: Edmonds [1965]
- since then, tons of research and still active
- many models of computation:
  monotone circuits, extended formulations,
  parallel, distributed, streaming/sublinear, ...

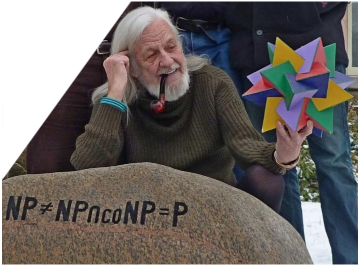


Perfect matching problem

Benchmark problem in computer science

Algorithms:

- bipartite: Jacobi [XIX century, weighted!]
- general: Edmonds [1965]
- since then, tons of research
  and still active
- many models of computation:
  monotone circuits, extended formulations,
  parallel, distributed, streaming/sublinear, ...



Parallel complexity

Class $\mathcal{NC}$: problems that paralellize completely

poly n processors

poly log n time

Parallel complexity

Class $\mathcal{NC}$: problems that paralellize completely

poly n processors

poly log n time

Main open question: is matching in $\mathcal{NC}$?

Parallel complexity

Class $\mathcal{NC}$: problems that paralellize completely

poly n processors

it's in **RANDOMIZED** $\mathcal{NC}$

poly log n time

Main open question: is matching in $\mathcal{NC}$?

Parallel complexity

- Matching is in RANDOMIZED $\mathcal{NC}$ [Lovász 1979]: has randomized algorithm that uses:
  - polynomially many processors
  - polylog time

- Search version is in RANDOMIZED $\mathcal{NC}$:
  - [Karp, Upfal, Wigderson 1986]
  - [Mulmuley, Vazirani, Vazirani 1987]

Parallel complexity

- Matching is in RANDOMIZED $\mathcal{NC}$ [Lovász 1979]: has randomized algorithm that uses:
  - polynomially many processors
  - polylog time

- Search version is in RANDOMIZED $\mathcal{NC}$:
  - [Karp, Upfal, Wigderson 1986]
  - [Mulmuley, Vazirani, Vazirani 1987]

Can we derandomize all efficient computation?

Parallel complexity

- Matching is in RANDOMIZED $\mathcal{NC}$ [Lovász 1979]: has randomized algorithm that uses:
  - polynomially many processors
  - polylog time

- Search version is in RANDOMIZED $\mathcal{NC}$:
  - [Karp, Upfal, Wigderson 1986]
  - [Mulmuley, Vazirani, Vazirani 1987]

Can we derandomize all efficient computation?

Can we derandomize one of these algorithms?

Is matching in $\mathcal{NC}$?

Yes, for restricted graph classes:

- bipartite regular [Lev, Pippenger, Valiant 1981]
- bipartite convex [Dekel, Sahni 1984]
- incomparability graphs [Kozen, Vazirani, Vazirani 1985]
- bipartite graphs with small number of perfect matchings [Grigoriev, Karpinski 1987]
- claw-free [Chrobak, Naor, Novick 1989]
- $K_{3,3}$-free (decision version) [Vazirani 1989]
- planar bipartite [Miller, Naor 1989]
- dense [Dahlhaus, Hajnal, Karpinski 1993]
- strongly chordal [Dahlhaus, Karpinski 1998]
- P_4-tidy [Parfenoff 1998]
- bipartite small genus [Mahajan, Varadarajan 2000]
- graphs with small number of perfect matchings [Agrawal, Hoang, Thierauf 2006]
- planar (search version) [Anari, Vazirani 2017]

Is matching in $\mathcal{NC}$?

Yes, for restricted graph classes:

- bipartite regular [Lev, Pippenger, Valiant 1981]
- bipartite convex [Dekel, Sahni 1984]
- incomparability graphs [Kozen, Vazirani, Vazirani 1985]
- bipartite graphs with small number of perfect matchings [Grigoriev, Karpinski 1987]
- claw-free [Chrobak, Naor, Novick 1989]
- $K_{3,3}$-free (decision version) [Vazirani 1989]
- planar bipartite [Miller, Naor 1989]
- dense [Dahlhaus, Hajnal, Karpinski 1993]
- strongly chordal [Dahlhaus, Karpinski 1998]
- P_4-tidy [Parfenoff 1998]
- bipartite small genus [Mahajan, Varadarajan 2000]
- graphs with small number of perfect matchings [Agrawal, Hoang, Thierauf 2006]
- planar (search version) [Anari, Vazirani 2017]

but not known for:

- general

Is matching in $\mathcal{NC}$?

Yes, for restricted graph classes:

- bipartite regular [Lev, Pippenger, Valiant 1981]
- bipartite convex [Dekel, Sahni 1984]
- incomparability graphs [Kozen, Vazirani, Vazirani 1985]
- bipartite graphs with small number of perfect matchings [Grigoriev, Karpinski 1987]
- claw-free [Chrobak, Naor, Novick 1989]
- $K_{3,3}$-free (decision version) [Vazirani 1989]
- planar bipartite [Miller, Naor 1989]
- dense [Dahlhaus, Hajnal, Karpinski 1993]
- strongly chordal [Dahlhaus, Karpinski 1998]
- P_4-tidy [Parfenoff 1998]
- bipartite small genus [Mahajan, Varadarajan 2000]
- graphs with small number of perfect matchings [Agrawal, Hoang, Thierauf 2006]
- planar (search version) [Anari, Vazirani 2017]

but not known for:

- general
- bipartite

Is matching in $\mathcal{NC}$?

Fenner, Gurjar and Thierauf [2015] showed:

- Bipartite matching is in QUASI-$\mathcal{NC}$
  ($n^{\text{poly} \log n}$ processors, poly log n time, deterministic)

Is matching in $\mathcal{NC}$?

Fenner, Gurjar and Thierauf [2015] showed:

- Bipartite matching is in QUASI-$\mathcal{NC}$
  ($n^{\text{poly} \log n}$ processors, $\text{poly} \log n$ time, deterministic)



- Approach fails for non-bipartite graphs

We show: general matching is in QUASI-$\mathcal{NC}$:

- $n^{\text{poly}\log n}$ processors
- $\text{poly}\log n$ time
- deterministic

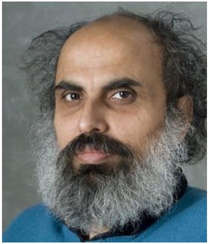
Outline

1. Isolating weight functions
   [Mulmuley, Vazirani, Vazirani 1987]

2. Bipartite case
   [Fenner, Gurjar, Thierauf 2015]

3. Difficulties of general case
   & our approach

1. Isolating weight functions [Mulmuley, Vazirani, Vazirani 1987]



Isolating weight functions

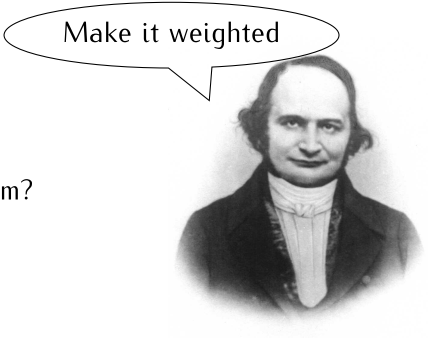
How to solve unweighted problem?

Isolating weight functions

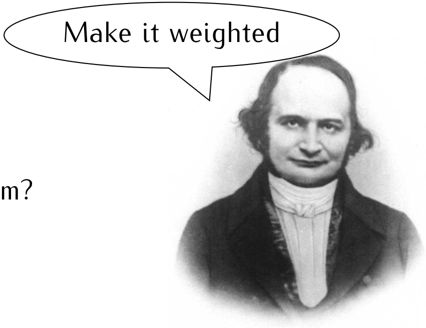
How to solve unweighted problem?

Isolating weight functions

How to solve unweighted problem?

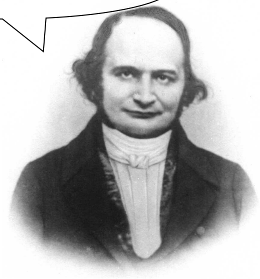


How to solve unweighted problem?



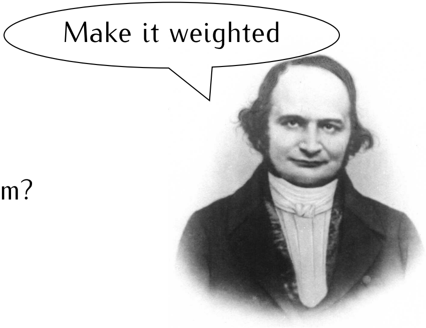
Isolating weight functions

Make it weighted



How to solve unweighted problem?

But **we** choose the weight function – do it smartly!

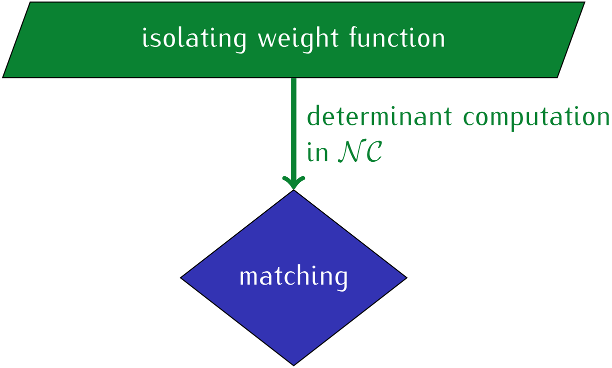


How to solve unweighted problem?

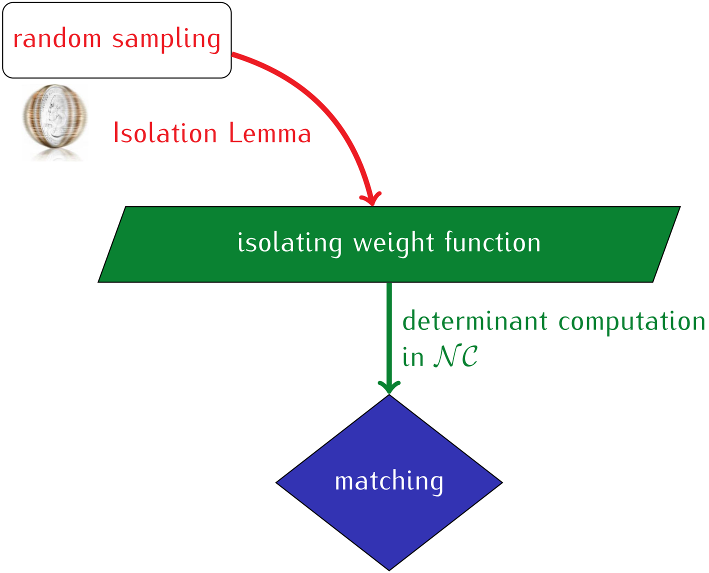
But **we** choose the weight function – do it smartly!

Weight function $w : E \to \mathbb{Z}_+$ is **isolating**
if there is a **unique** min-weight perfect matching

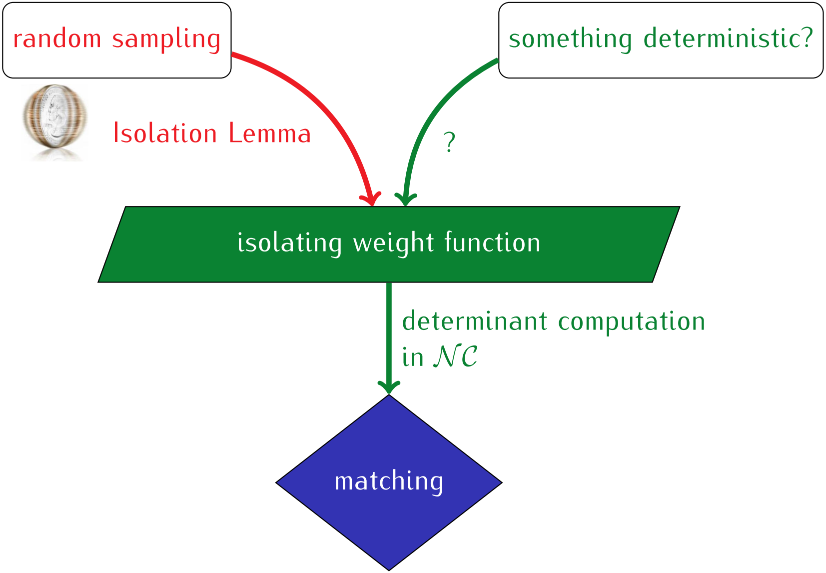
[Mulmuley, Vazirani, Vazirani 1987]



[Mulmuley, Vazirani, Vazirani 1987]



[Mulmuley, Vazirani, Vazirani 1987]



Isolation Lemma

Weight function $w : E \to \mathbb{Z}_+$ is **isolating**
if there is a **unique** min-weight perfect matching

Isolation Lemma [MVV 1987]

If each $w(e)$ picked randomly from $\{1, 2, ..., n^3\}$,
then $\mathrm{P}[w \text{ isolating}] \geq 1 - \frac{1}{n}$

Isolation Lemma

Weight function $w : E \to \mathbb{Z}_+$ is **isolating**
if there is a **unique** min-weight perfect matching

Isolation Lemma [MVV 1987]

If each $w(e)$ picked randomly from $\{1, 2, ..., n^3\}$,
then $\mathrm{P}[w \text{ isolating}] \geq 1 - \frac{1}{n}$

- holds more generally,
  for any set family in place of matchings!
- many applications in complexity theory

Derandomize the Isolation Lemma

- **Challenge**:
  get an isolating weight function
  deterministically in $\mathcal{NC}$

- **We prove**:
  can construct $n^{O(\log^2 n)}$ weight functions in QUASI-$\mathcal{NC}$
  such that one of them is isolating

- We do it without looking at the graph

- **Implies: matching is in QUASI-$\mathcal{NC}$**

Derandomize the Isolation Lemma

- **Challenge**:
  get an isolating weight function
  deterministically in $\mathcal{NC}$
- **We prove**:
  can construct $n^{O(\log^2 n)}$ weight functions in QUASI-$\mathcal{NC}$
  such that one of them is isolating
- We do it without looking at the graph
- **Implies: matching is in QUASI-$\mathcal{NC}$**

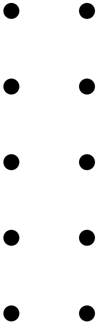
2. Bipartite case
[Fenner, Gurjar, Thierauf 2015]

Goal: how to construct $n^{O(\log n)}$ weight functions such that one of them is isolating?

Isolating matchings

What if w is **not** isolating?

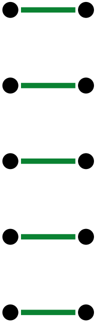
- there are perfect matchings M, M' with $w(M) = w(M')$ minimum



Isolating matchings

What if w is **not** isolating?

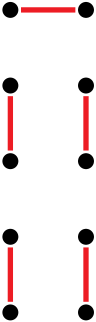
- there are perfect matchings M, M' with $w(M) = w(M')$ minimum



Isolating matchings

What if w is **not** isolating?

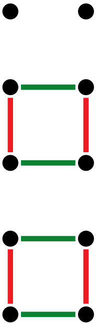
- there are perfect matchings M, M' with $w(M) = w(M')$ minimum



Isolating matchings

What if w is **not** isolating?

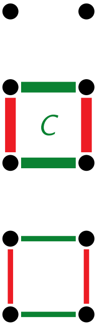
- there are perfect matchings M, M' with $w(M) = w(M')$ minimum
- symmetric difference = alternating cycles



Isolating matchings

What if w is **not** isolating?

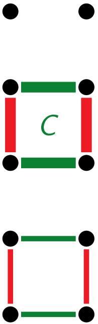
- there are perfect matchings M, M' with $w(M) = w(M')$ minimum
- symmetric difference = alternating cycles
- in each cycle C, $w(\text{GREEN}) = w(\text{RED})$ (otherwise could get lighter matching)



Isolating matchings

What if w is **not** isolating?

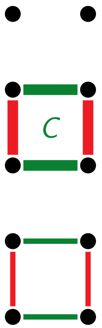
- there are perfect matchings M, M' with $w(M) = w(M')$ minimum
- symmetric difference = alternating cycles
- in each cycle C, $w(\text{GREEN}) = w(\text{RED})$ (otherwise could get lighter matching)
- define **discrepancy** of a cycle: $d_w(C) := w(\text{GREEN}) - w(\text{RED})$



Isolating matchings

What if w is **not** isolating?

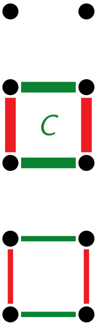
- there are perfect matchings M, M' with $w(M) = w(M')$ minimum
- symmetric difference = alternating cycles
- in each cycle C, $w(\text{GREEN}) = w(\text{RED})$ (otherwise could get lighter matching)
- define **discrepancy** of a cycle: $d_w(C) := w(\text{GREEN}) - w(\text{RED})$
- $d_w(C) = 0$



Isolating matchings

What if w is **not** isolating?

- there are perfect matchings M, M' with $w(M) = w(M')$ minimum
- symmetric difference = alternating cycles
- in each cycle C, $w(\text{GREEN}) = w(\text{RED})$ (otherwise could get lighter matching)
- define **discrepancy** of a cycle: $d_w(C) := w(\text{GREEN}) - w(\text{RED})$
- $d_w(C) = 0$

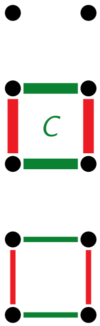


If $(\forall C)\ d_w(C) \neq 0$, then w isolating!

Isolating matchings

What if w is **not** isolating?

- there are perfect matchings M, M' with $w(M) = w(M')$ minimum
- symmetric difference = alternating cycles
- in each cycle C, $w(\text{GREEN}) = w(\text{RED})$ (otherwise could get lighter matching)
- define **discrepancy** of a cycle: $d_w(C) := w(\text{GREEN}) - w(\text{RED})$
- $d_w(C) = 0$



If $(\forall C)\ d_w(C) \neq 0$, then w isolating!

New objective: assign $\neq 0$ discrepancy to every cycle

Removing cycles

New objective: assign $\neq 0$ discrepancy to every cycle

Removing cycles

New objective: assign $\neq 0$ discrepancy to every cycle

Lemma

For any n^4 cycles,
can find a weight function w that
assigns **all of them** $\neq 0$ discrepancy.

Removing cycles

New objective: assign $\neq 0$ discrepancy to every cycle

Lemma

For any n^4 cycles,
can find a weight function w that
assigns **all of them** $\neq 0$ discrepancy.

Removing cycles

New objective: assign $\neq 0$ discrepancy to every cycle

Lemma

For any n^4 cycles,
can find a weight function w that
assigns **all of them** $\neq 0$ discrepancy.

If $\leq n^4$ cycles in the graph: done!

Removing cycles

New objective: assign $\neq 0$ discrepancy to every cycle

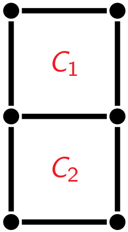
Lemma

For any n^4 cycles,
can find a weight function w that
assigns **all of them** $\neq 0$ discrepancy.

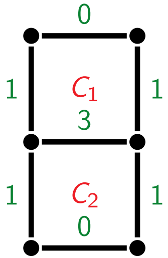
If $\leq n^4$ cycles in the graph: done!

Not so easy, but we can cope with all **4-cycles**.

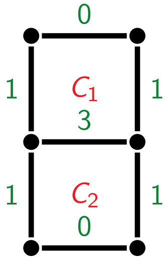
Removing cycles



Removing cycles



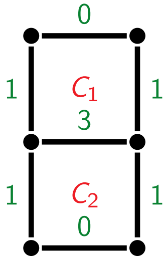
Removing cycles



$d_w(C_1) = 1 \neq 0$
$d_w(C_2) = 1 \neq 0$

Removing cycles

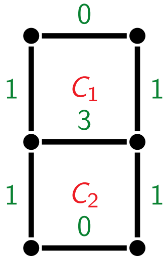
Active subgraph:
those edges that are in a min-weight perfect matching



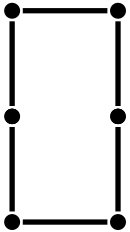
$d_w(C_1) = 1 \neq 0$
$d_w(C_2) = 1 \neq 0$
$\Longrightarrow$

Removing cycles

Active subgraph:
those edges that are in a min-weight perfect matching



$d_w(C_1) = 1 \neq 0$
$d_w(C_2) = 1 \neq 0$
$\Longrightarrow$

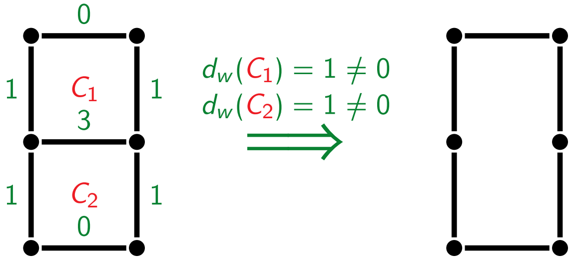


Removing cycles

Active subgraph:
those edges that are in a min-weight perfect matching

Bipartite key property

Once we assign a cycle $\neq 0$ discrepancy,
it will disappear from the active subgraph.

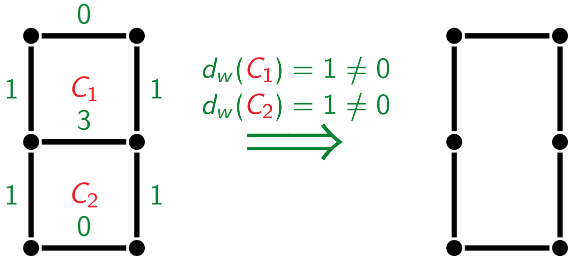


Removing cycles

Active subgraph:
those edges that are in a min-weight perfect matching

Bipartite key property

Once we assign a cycle $\neq 0$ discrepancy,
it will disappear from the active subgraph.

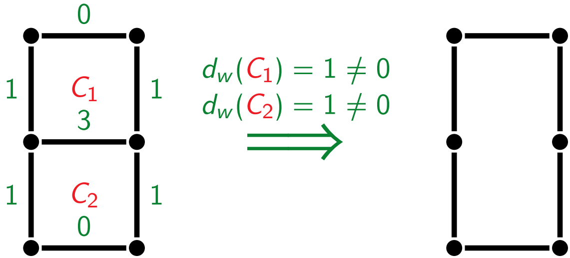


Removing cycles

Active subgraph:
those edges that are in a min-weight perfect matching

Bipartite key property

Once we assign a cycle $\neq 0$ discrepancy,
it will disappear from the active subgraph.



By assigning $\neq 0$ discrepancy to 4-cycles, we can remove them.
Then continue restricted to the smaller active subgraph!

Isolating in stages

Crucial idea:

- Can find w_1 such that 4-cycles are assigned $\neq 0$ discrepancy

Isolating in stages

Crucial idea:

- Can find w_1 such that 4-cycles are removed from active subgraph

Isolating in stages

Crucial idea:

- Can find w_1 such that 4-cycles are removed from active subgraph
- Can find w_2 such that (≤ 8)-cycles are removed from active subgraph
- Can find w_3 such that (≤ 16)-cycles are removed from active subgraph
- ...
- Can find $w_{\log n}$ such that all cycles are removed from active subgraph $\implies$ done!

Isolating in stages

Crucial idea:

- Can find w_1 such that 4-cycles are removed from active subgraph
- Can find w_2 such that (≤ 8)-cycles are removed from active subgraph
- Can find w_3 such that (≤ 16)-cycles are removed from active subgraph
- ...
- Can find $w_{\log n}$ such that all cycles are removed from active subgraph $\Longrightarrow$ done!

Isolating in stages

Crucial idea:

- Can find w_1 such that 4-cycles are removed from active subgraph
- Can find w_2 such that (≤ 8)-cycles are removed from active subgraph
- Can find w_3 such that (≤ 16)-cycles are removed from active subgraph
- ...
- Can find $w_{\log n}$ such that all cycles are removed from active subgraph $\Longrightarrow$ done!

Isolating in stages

Crucial idea:

- Can find w_1 such that 4-cycles are removed from active subgraph
- Can find w_2 such that (≤ 8)-cycles are removed from active subgraph
- Can find w_3 such that (≤ 16)-cycles are removed from active subgraph
- ...
- Can find $w_{\log n}$ such that all cycles are removed from active subgraph $\implies$ done!

Isolating in stages

Crucial idea:

- Can find w_1 such that 4-cycles are removed from active subgraph
- Can find w_2 such that (≤ 8)-cycles are removed from active subgraph
- Can find w_3 such that (≤ 16)-cycles are removed from active subgraph
- ...
- Can find $w_{\log n}$ such that all cycles are removed from active subgraph $\implies$ done!

Actually, not sure how to find in $\mathcal{NC}$ some w_i that is good...
But always some w_i of a special form is good.
Try all combinations $(w_1, w_2, \ldots, w_{\log n})$ **obliviously**!
There are $n^{O(\log n)}$ many.

3. Difficulties of general case & our approach

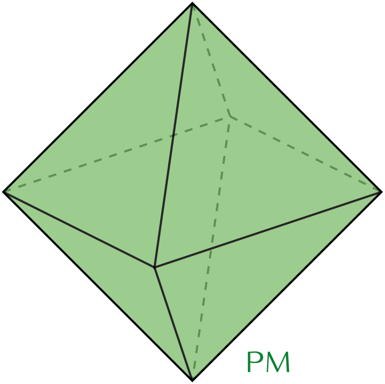
Bipartite key property fails

Bipartite key property

~~Once we assign a cycle $\neq 0$ discrepancy, it will disappear from the active subgraph.~~

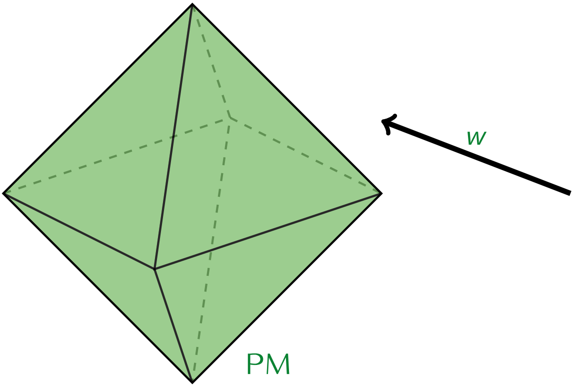
Polyhedral perspective

- PM: perfect matching polytope
  (convex hull of all perfect matchings)



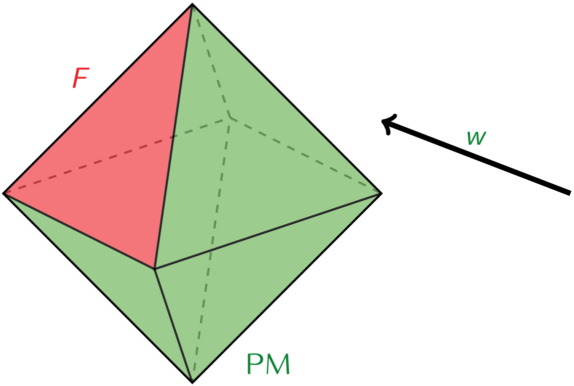
Polyhedral perspective

- PM: perfect matching polytope
  (convex hull of all perfect matchings)



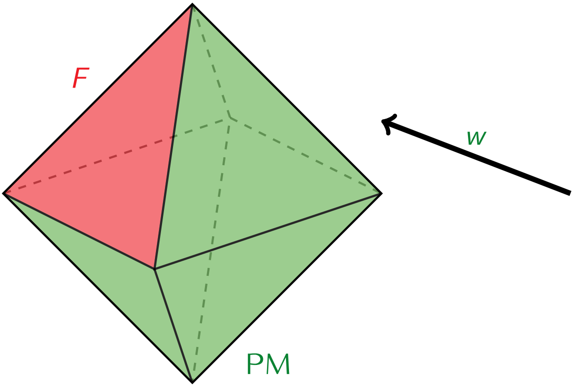
Polyhedral perspective

- PM: perfect matching polytope
  (convex hull of all perfect matchings)
- F: set of points in PM that minimize w
  - F is a face of PM



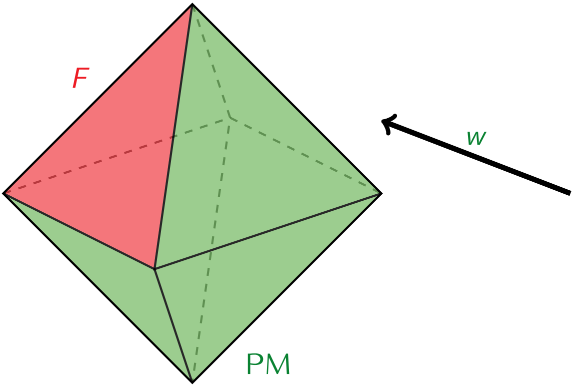
Polyhedral perspective

- PM: perfect matching polytope (convex hull of all perfect matchings)
- F: set of points in PM that minimize w
  - F is a face of PM
- w isolating $\iff |F| = 1$ (F is a vertex)



Polyhedral perspective

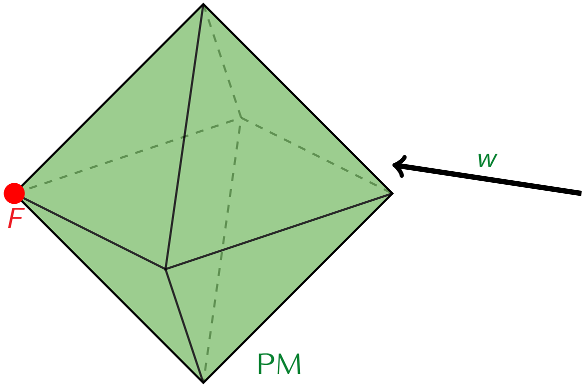
- PM: perfect matching polytope
  (convex hull of all perfect matchings)
- F: set of points in PM that minimize w
  - F is a face of PM
- w isolating $\iff |F| = 1$ (F is a vertex)



w not isolating

Polyhedral perspective

- PM: perfect matching polytope (convex hull of all perfect matchings)
- F: set of points in PM that minimize w
  - F is a face of PM
- w isolating $\iff |F| = 1$ (F is a vertex)



w isolating

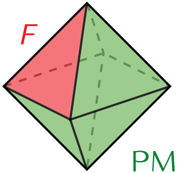
LP formulation

Edmonds [1965]

PM described as set of $x \in \mathbb{R}^E$ such that:

- $x_e \geq 0$ for every edge e
- $x(\delta(v)) = 1$ for every vertex v
- $x(\delta(S)) \geq 1$ for every odd set S of vertices

($\delta(S)$ = edges crossing S)



LP formulation

Edmonds [1965]

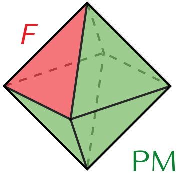
PM described as set of $x \in \mathbb{R}^E$ such that:

- $x_e \geq 0$ for every edge e
- $x(\delta(v)) = 1$ for every vertex v
- $x(\delta(S)) \geq 1$ for every odd set S of vertices

($\delta(S)$ = edges crossing S)

So every face F is given as:

$$F = \{x \in \text{PM} : x_e = 0 \quad \text{for some edges } e, \quad x(\delta(S)) = 1 \quad \text{for some odd sets } S\}$$



LP formulation

Edmonds [1965]

PM described as set of $x \in \mathbb{R}^E$ such that:

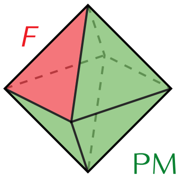
- $x_e \geq 0$ for every edge e
- $x(\delta(v)) = 1$ for every vertex v
- $x(\delta(S)) \geq 1$ for every odd set S of vertices

($\delta(S)$ = edges crossing S)

So every face F is given as:

$$F = \{x \in \text{PM} : x_e = 0 \quad \text{for some edges } e, \quad x(\delta(S)) = 1 \quad \text{for some odd sets } S\}$$

- In bipartite case:
  $F = \{x \in \text{PM} : \ x_e = 0 \ \text{ for some edges } e\}$
  (F given by the active subgraph)
- Now, faces are exponentially harder
- Need $2^{\Omega(n)}$ inequalities [Rothvoss 2013]



Edmonds [1965]

PM described as set of $x \in \mathbb{R}^E$ such that:

- $x_e \geq 0$ for every edge e
- $x(\delta(v)) = 1$ for every vertex v

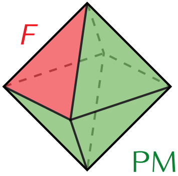
($\delta(S)$ = edges crossing S)

$x(\delta(S)) \geq 1$ for every odd set S of vertices

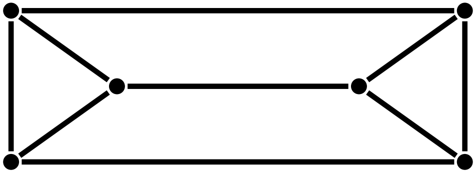
 **Bipartite key property fails!** 

$x(\delta(S)) = 1$ for some odd sets $S\}$

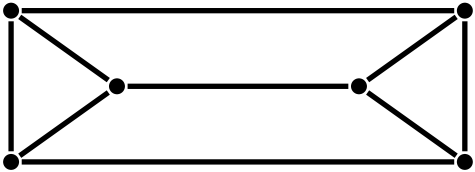
- In bipartite case:
  $F = \{x \in \text{PM} : \ x_e = 0$ for some edges $e\}$
  (F given by the active subgraph)
- Now, faces are exponentially harder
- Need $2^{\Omega(n)}$ inequalities [Rothvoss 2013]



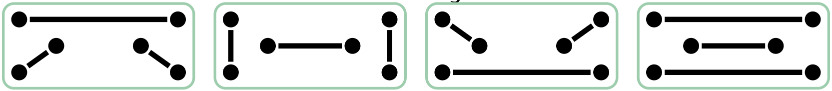
How bipartite key property fails



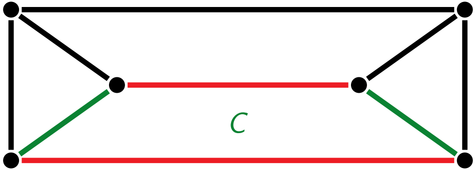
How bipartite key property fails



PM: convex hull of all four matchings:

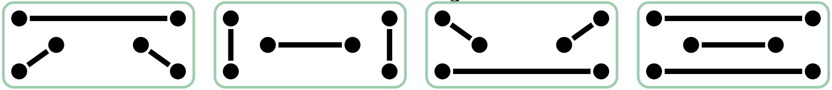


How bipartite key property fails

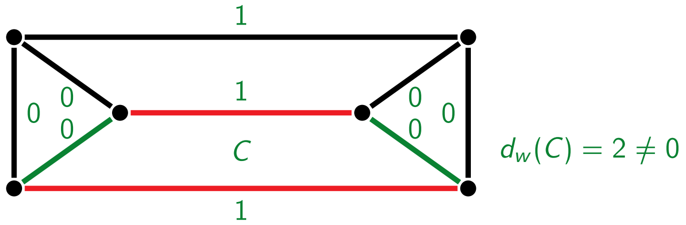


want:
$d_w(C) \neq 0$

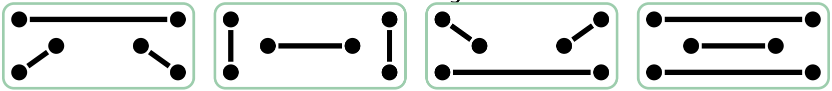
PM: convex hull of all four matchings:



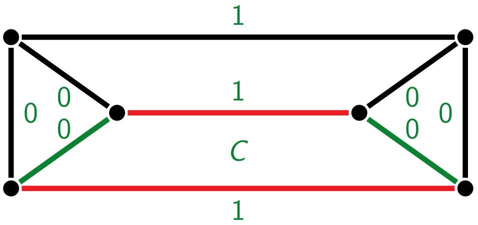
How bipartite key property fails



PM: convex hull of all four matchings:

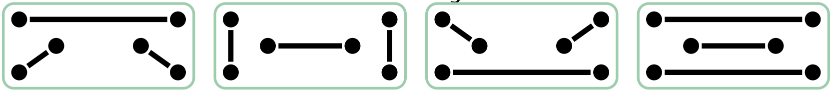


How bipartite key property fails

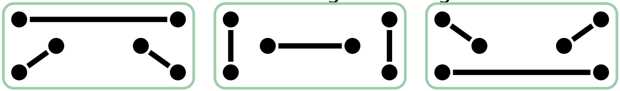


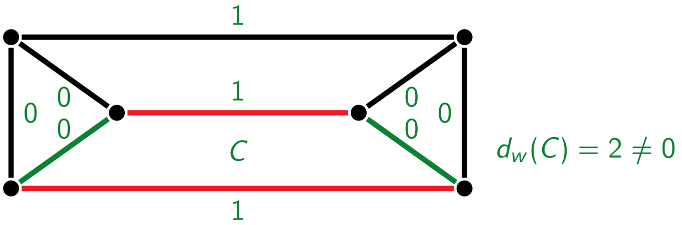
$d_w(C) = 2 \neq 0$

PM: convex hull of all four matchings:

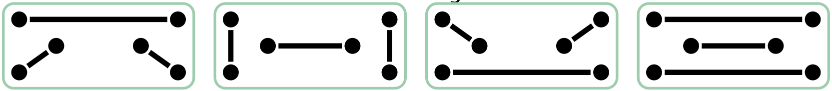


F: convex hull of matchings of weight 1:

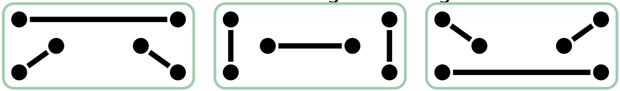




PM: convex hull of all four matchings:

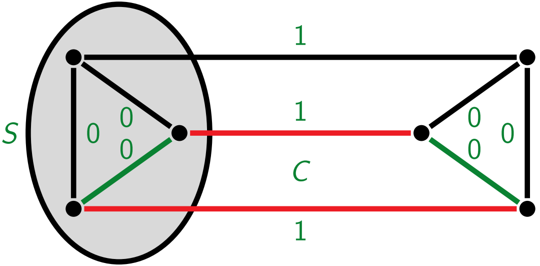


F: convex hull of matchings of weight 1:



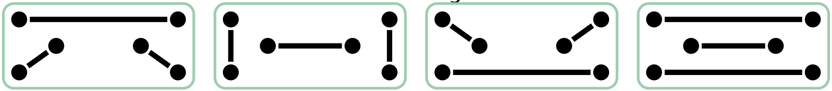
$F \subsetneq PM$ but still has all edges...

How bipartite key property fails

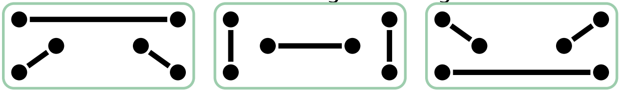


$d_w(C) = 2 \neq 0$

PM: convex hull of all four matchings:



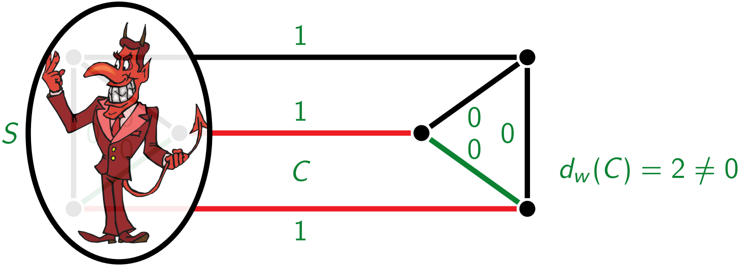
F: convex hull of matchings of weight 1:



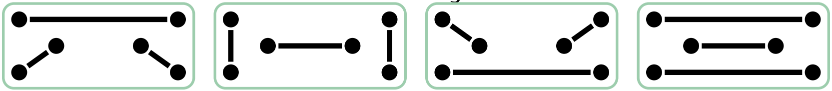
$F \subsetneq PM$ but still has all edges...

$F = \{x \in PM : x(\delta(S)) = 1\}$

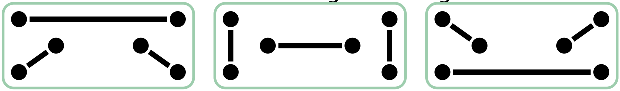
How bipartite key property fails



PM: convex hull of all four matchings:



F: convex hull of matchings of weight 1:

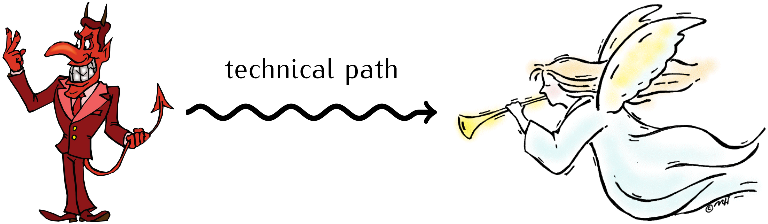


$F \subsetneq PM$ but still has all edges...

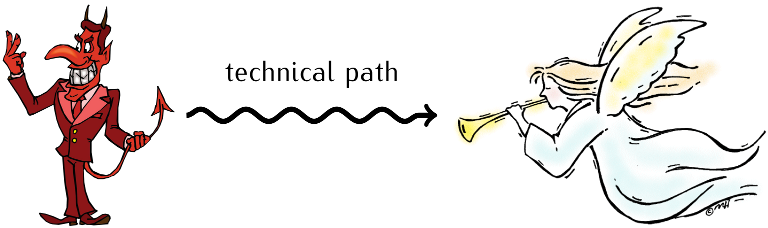
$F = \{x \in PM : x(\delta(S)) = 1\}$

How we cope

How we cope



How we cope



Main ingredients:

- Laminar family of tight cut constraints
- Tight cut constraints decompose the instance
  - $\Rightarrow$ divide-and-conquer approach

Laminarity

Every face F is given as:

$$F = \{x \in \mathrm{PM} : \begin{array}{ll} x_e = 0 & \text{for some edges } e, \\ x(\delta(S)) = 1 & \text{for some odd sets } S \}\end{array}$$

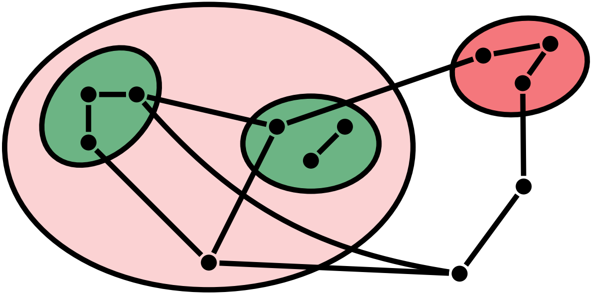
Laminarity

Every face F is given as:

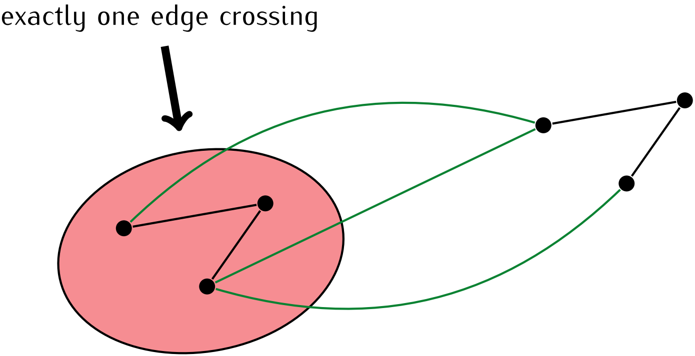
$$F = \{x \in \mathrm{PM} : x_e = 0 \quad \text{for some edges } e, \\ x(\delta(S)) = 1 \quad \text{for some odd sets } S\}$$

Great news: "some" can be chosen to be a laminar family!

(at most $n/2$ constraints instead of exponentially many to describe a face)



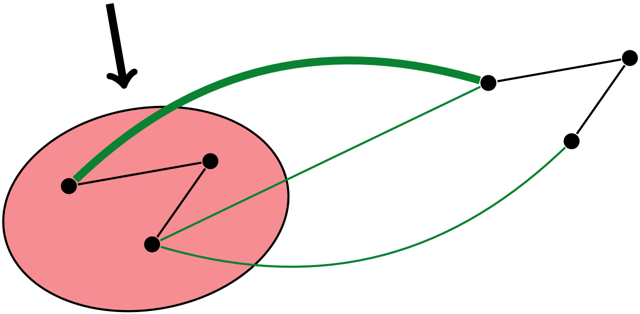
Tight odd cuts are not all bad



- once we fix a boundary edge...

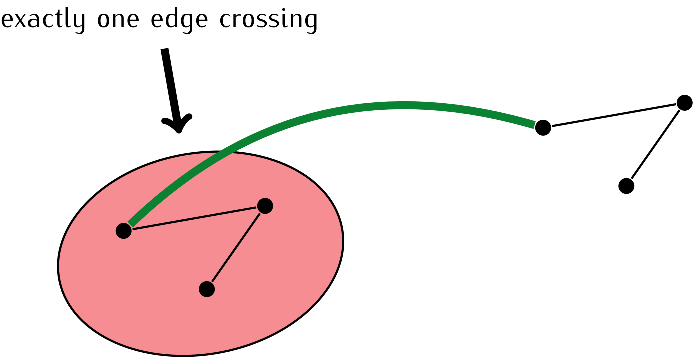
Tight odd cuts are not all bad

exactly one edge crossing



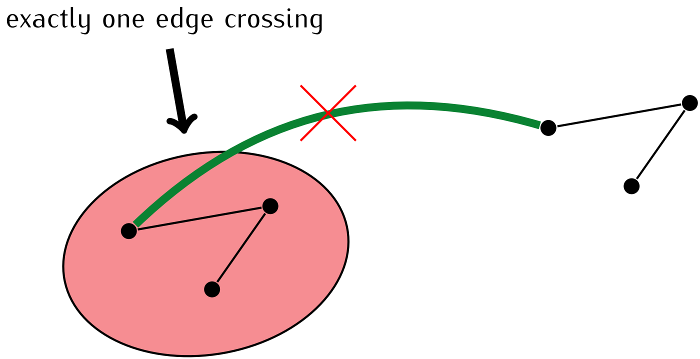
- once we fix a boundary edge...

Tight odd cuts are not all bad



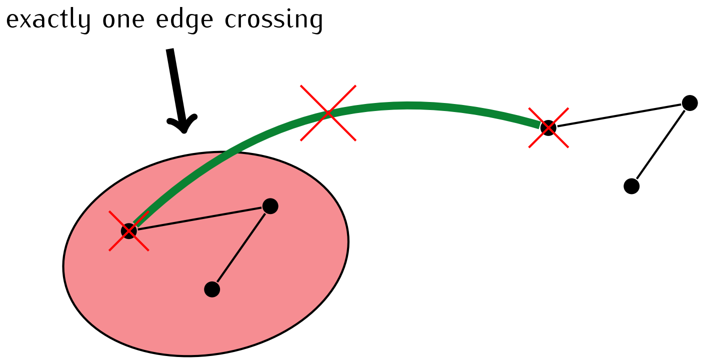
- once we fix a boundary edge...

Tight odd cuts are not all bad



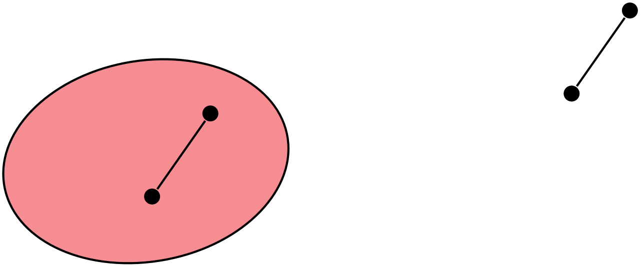
- once we fix a boundary edge...

Tight odd cuts are not all bad



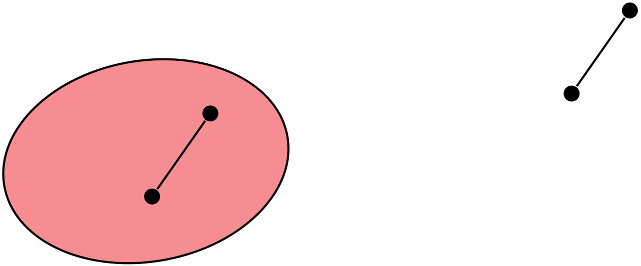
- once we fix a boundary edge...

Tight odd cuts are not all bad

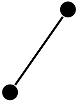


- once we fix a boundary edge...

Tight odd cuts are not all bad



- once we fix a boundary edge...
- ... the instance decomposes into two **independent** ones

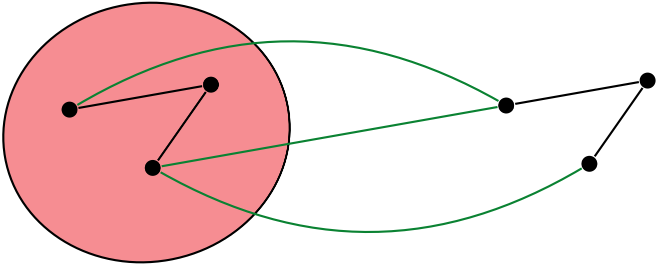


- once we fix a boundary edge...
- ... the instance decomposes into two **independent** ones

Divide & conquer

Simplest case of laminar family: only one tight odd set

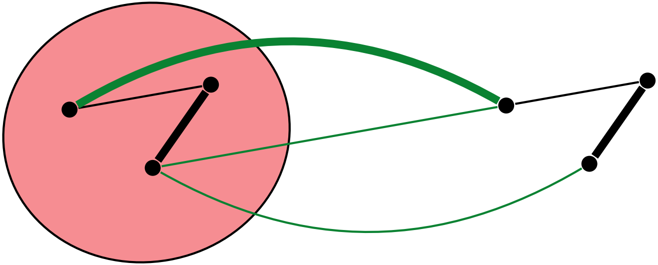
Between friends: cycles that do not cross tight odd sets behave like in the bipartite case and can thus be removed



Divide & conquer

Simplest case of laminar family: only one tight odd set

Between friends: cycles that do not cross tight odd sets behave like in the bipartite case and can thus be removed

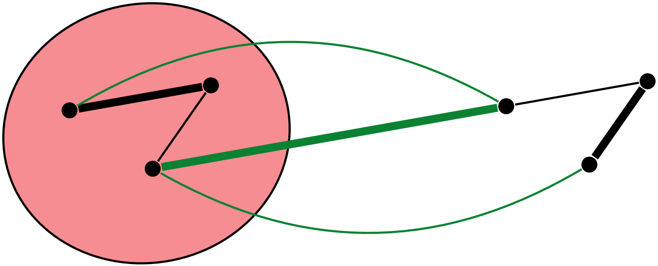


- then every boundary edge determines entire matching

Divide & conquer

Simplest case of laminar family: only one tight odd set

Between friends: cycles that do not cross tight odd sets behave like in the bipartite case and can thus be removed

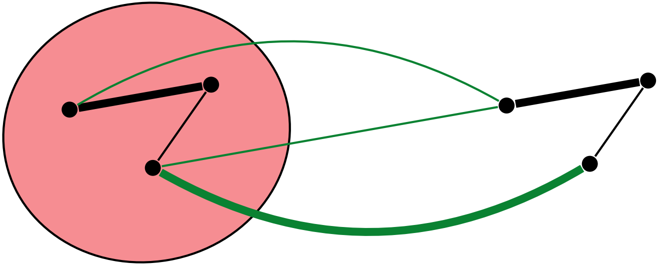


- then every boundary edge determines entire matching

Divide & conquer

Simplest case of laminar family: only one tight odd set

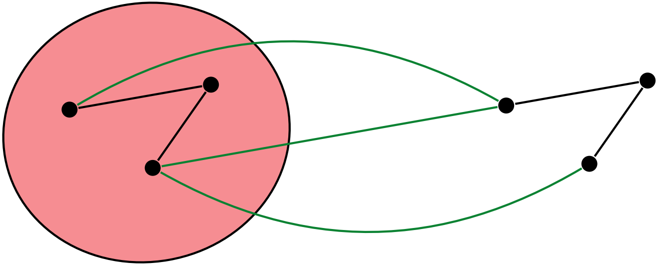
Between friends: cycles that do not cross tight odd sets behave like in the bipartite case and can thus be removed



- then every boundary edge determines entire matching

Simplest case of laminar family: only one tight odd set

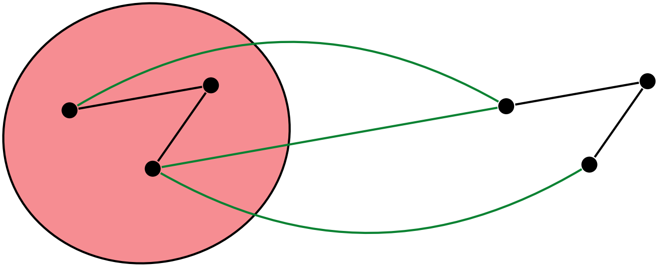
Between friends: cycles that do not cross tight odd sets behave like in the bipartite case and can thus be removed



- then every boundary edge determines entire matching
- so: at most n^2 perfect matchings

Simplest case of laminar family: only one tight odd set

Between friends: cycles that do not cross tight odd sets behave like in the bipartite case and can thus be removed



- then every boundary edge determines entire matching
- so: at most n^2 perfect matchings
- easy to isolate

Our dichotomy

Dichotomy:

- remove cycles not crossing tight odd-sets
- use tight odd-sets to decompose problem (divide & conquer)

Our dichotomy

Dichotomy:

- remove cycles not crossing tight odd-sets
- use tight odd-sets to decompose problem (divide & conquer)

Details: see paper or talk to me :)

Future work

- go down to $\mathcal{NC}$
  - even for bipartite graphs
  - ✓ for planar graphs: [Anari, Vazirani 2017]

Future work

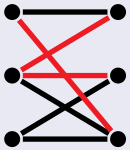
- go down to $\mathcal{NC}$
  - even for bipartite graphs
  - ✓ for planar graphs: [Anari, Vazirani 2017]
- derandomize Isolation Lemma in other cases
  - ✓ matroid intersection: [Gurjar, Thierauf 2017]
  - ✓ totally unimodular polytopes: [Gurjar, Thierauf, Vishnoi 2017]
  - any efficiently solvable 0/1-polytope?

Future work

- go down to $\mathcal{NC}$
  - even for bipartite graphs
  - ✓ for planar graphs: [Anari, Vazirani 2017]

- derandomize Isolation Lemma in other cases
  - ✓ matroid intersection: [Gurjar, Thierauf 2017]
  - ✓ totally unimodular polytopes: [Gurjar, Thierauf, Vishnoi 2017]
  - any efficiently solvable 0/1-polytope?

EXACT MATCHING



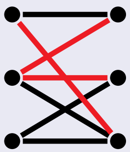
Given: graph with some edges red, number k.
Is there a perfect matching with exactly k red edges?

- randomized complexity: even RANDOMIZED $\mathcal{NC}$
- deterministic complexity: is it in $\mathcal{P}$?

Future work

- go down to $\mathcal{NC}$
  - even for bipartite graphs
  - ✓ for planar graphs: [Anari, Vazirani 2017]
- derandomize Isolation Lemma in other cases
  - ✓ matroid intersection: [Gurjar, Thierauf 2017]
  - ✓ totally unimodular polytopes: [Gurjar, Thierauf, Vishnoi 2017]
  - any efficiently solvable 0/1-polytope?

EXACT MATCHING



Given: graph with some edges red, number k.
Is there a perfect matching with exactly k red edges?

- randomized complexity: even RANDOMIZED $\mathcal{NC}$
- deterministic complexity: is it in $\mathcal{P}$?

Thank you!